Name of College - S-S- College S-bug.

Deptt - Mathematics

<u>Topic</u> - Method For Finding the oblique Asymptote

Class - B-Sc

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<u>Method</u> For Finding the oblique Asymptotes

Let the Asymptote be $y = mx + c$

Let the Equation of the Curve be

$$g_n(x,y) + g_{n-1}(x,y) + \cdots + g_1(x,y) + k = 0$$

Where $g_n(x,y)$ denotes the Term of highest degree of the curve.

STEP I - Put $x=1$, $y=m$ in
$g_n(x,y)$, $g_{n-1}(x,y)$, ..., $g_1(x,y)$

STEP II - Find all the real roots of $g_n(m) = 0$

STEP III - If m is non-repeated root, then corresponding value of c is given by

$$c\, g_n'(m) + g_{n-1}(m) = 0 \quad [g_n'(m) \neq 0]$$

If $g_n'(m) = 0$ then there is no

Asymptote to the curve corresponding to this value of m

STEP IV — If m is repeated root then the two values of C occurring twice are given by

$$\frac{c^2}{L2} \phi''_n(m) + \frac{c}{L1} \phi'_{n-1}(m) + \phi_{n-2}(m) = 0$$

$(\phi''_n(m) \neq 0)$

STEP 5 :- The Asymptote of the curve is

$$y = mx + c$$

Ex:→ Find all the Asymptotes of the curve

$$x^3 - x^2y - xy^2 + y^3 + 2x^2 - 4y^2 + 2xy + x + y + 1 = 0$$

Solution → In the curve the highest degree terms of x is x^3 and its co-efficient is 1

Equating to zero we get

1 = 0 which is absurd

This curve has no Asymptote parallel to x-axis.

Also the co-efficient of highest degree term of y is 1

Equating to zero

1 = 0 is also absurd

⇒ This this curve has no Asymptote parallel to y-axis

Thus Hence we find

The curve has no any Asymptotes parallel to x-axis or parallel to y-axis.

Now For finding oblique Asymptotes i.e. Asymptotes which are neither parallel to x-axis nor parallel to y-axis

Here $g_3(x,y) = x^3 - x^2y + y^3 - xy^2$

$g_2(x,y) = 2x^2 - 4y^2 + 2xy$.

$g_1(x,y) = x + y$.

Let the Asymptote be given by

$y = mx + c$

STEP I put $x=1$, $m=y$ in $g_3(x,y)$

$\therefore g_3(m) = 1 - m + m^3 - m^2$

$g_2(m) = 2 - 4m^2 + 2m$.

$g_1(m) = 1 + m$

STEP 2: $\rightarrow$ The values of m are obtained Now by $g_3(m) = 0$

$\Rightarrow m^3 - m^2 - m + 1 = 0$

$\Rightarrow m^2(m-1) - (m-1) = 0$

$\Rightarrow m^2 - 1 (m-1) = 0 \Rightarrow m = \pm 1$, Here $m = 1, 1, -1$

Here $m = -1$ is a non-repeated root and

$m = 1$ is a repeated root.

STEP III for $m = -1$ (non-repeated root)

The corresponding value of c is given by

$$c\, g_3'(m) + g_2(m) = 0$$

here $g_3'(m) = 3m^2 - 2m - 1 = 0$

$$\therefore c(3m^2 - 2m - 1) + (2 + 2m - 4m^2) = 0$$

$$\Rightarrow c(3m^2 - 2m - 1) = 4m^2 - 2m - 2$$

$$\Rightarrow c = \frac{4m^2 - 2m - 2}{3m^2 - 2m - 1} \quad \text{put } m = -1$$

$$= \frac{4 + 2 - 2}{3 + 2 - 1} = \frac{4}{4} = 1$$

Thus for $m = -1$ $c = 1$

STEP 4 → for $m = 1$ (Repeated Root),

the value of c is given by

$$\frac{c^2}{\lfloor 2} g_3''(m) + \frac{c}{\lfloor 1} g_2'(m) + g_1(m) = 0$$

$\because g_3(m) = m^3 - m^2 - m + 1 = 0$ $\quad g_3'(m) = 3m^2 - 2m - 1$

$g_2(m) = 2 - 4m^2 + 2m$ $\quad g_2'(m)$ $\quad g_3''(m) = 6m - 2$

Putting these values in

$$\frac{c^2}{L2}\, \phi_3''(m) + \frac{c}{L1}\, \phi_2'(m) + \phi_1(m) = 0$$

$$\Rightarrow \frac{c^2}{L2}(6m-2) + \frac{c}{L1}(2-8m) + m+1 = 0$$

put $m=1$

$$\frac{c^2}{L2} \times \overset{2}{\cancel{4}} + \frac{c}{L1}(-6) + 2 = 0$$

$$2c^2 - 6c + 2 = 0$$

$$c^2 - 3c + 1 = 0$$

$$\therefore c = \frac{3 \pm \sqrt{9-4}}{2}$$

$$= \frac{3 \pm \sqrt{5}}{2}$$

When $m = 1$ $c = \frac{3+\sqrt{5}}{2}$

$m = 1$ $c = \frac{3-\sqrt{5}}{2}$

$m = -1$ $c = 1$

$\therefore$ Required Asymptotes are

$y = -x + 1 \Rightarrow x - y + 1 = 0$

$y = 1 \cdot x + \frac{3+\sqrt{5}}{2}$

$x - 2y + (3+\sqrt{5}) = 0$

$y = 1 \cdot x + \frac{3-\sqrt{5}}{2}$

$x - y - \frac{3-\sqrt{5}}{2} = 0$

Thus Required Asymptotes are

$x + y - 1 = 0$

$2(y - x) = 3 + \sqrt{5}$

$2(y - x) = 3 - \sqrt{5}$

Ex:→ Find the Asymptotes of the curve

$y^3 + x^2y + 2xy^2 - y + 1 = 0$

Solution :→ Equation of the Curve

$y^3 + x^2y + 2xy^2 - y + 1 = 0$

for Asymptotes parallel to y axis equating the co-efficient of $y^3 = 0$

i.e $1 = 0$ Which is Absurd

$\Rightarrow$ No any Asymptotes is parallel to y-axis.

for Asymptotes parallel to x-axis Equating to zero the co-efficient of x^2

Here equation of x^2 is y

$\Rightarrow$ $y = 0$ $\Rightarrow$ it is a asymptote parallel to x-axis.

~~Put x=1, y=m~~ ~~in the equation~~ of ~~curve~~

~~$m^3 + m + 2m^2$~~ $- m + 1 = 0$

~~$\Rightarrow m^3 + 2m^2 + 1 = 0$~~

for oblique Asymptotes:

$\phi_3(x,y) = y^3 + x^2y + 2xy^2$

$\phi_2(x,y) = 0$

$\phi_1(x,y) = -y$

$\therefore$ putting $x=1$, $y=m$ in $\phi_3(x,y)$

$\phi_3(m) = m^3 + m + 2m^2 \quad \therefore \phi_3'(m) = 3m^2 + 4m + 1$

$\phi_2(m) = 0$

$\phi_1(m) = -m.$

Now $\phi_3'(m) = 0$

$\cancel{3m^2 + 4m + 1} = 0$

The values of m are obtained by solving the equation

$\phi_3(m) = 0$

$m^3 + 2m^2 + m = 0$

$\Rightarrow m(m^2 + 2m + 1) = 0$

$\Rightarrow m = 0 \quad m = -1, -1$

for $m=0$ (non-repeating value) the value of c is given by

$$c = \frac{\phi_2(m)}{\phi_1'(m)} = \frac{0}{3m^2+4m+1} = 0$$

$\therefore y = 0 \cdot x + 0 \Rightarrow y = 0$ is the Asymptote.

for $m = -1, -1$ (Repeated root)

$$\frac{c^2}{L^2} \phi_3''(m) + \frac{c}{L_1} \phi_2'(m) + \phi_1(m) = 0$$

$$\Rightarrow \frac{c^2}{L^2}(6m+4) + 0 - m = 0$$

$$c^2(3m+2) - m = 0$$

$$\therefore c^2 = \frac{m}{3m+2} = \frac{-1}{-1} = 1$$

$$\therefore c = \pm 1$$

$\therefore$ ~~Asy~~ Asymptote

$y = -x + 1$ and $y = -x - 1$

$x + y + 1 = 0$

$\Rightarrow x + y - 1 = 0.$

Thus Required Asymptotes are

$x + y - 1 = 0$ }
$x + y + 1 = 0$ } oblique Asymptotes.

$y = 0.$ Asymptotes parallel to x-axis